AD-A041 738 NAVAL RESEARCH LAB WASHINGTON D C F/G 17/6
SUPERCONDUCTING MAGNETIC CALIBRATION SOURCE FOR MAGNETIC ANOMAL--ETC(U)
JUN 77 M NISENOFF, J J KENNEDY, A W WEBB
UNCLASSIFIED NRL-MR-3534 NL

| OF |

AD
A 041738

END
DATE
FILMED
8 — 77

12 B.S.

NRL Memorandum Report 3534

Superconducting Magnetic Calibration Source for Magnetic Anomaly Detection (MAD) Operations

M. NISENOFF, J. J. KENNEDY and A. W. WEBB

Material Sciences Division

June 1977

NAVAL RESEARCH LABORATORY
Washington, D.C.

Approved for public release: distribution unlimited.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NRL Memorandum Report 3534	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) 6 SUPERCONDUCTING MAGNETIC CALIBRATION SOURCE FOR MAGNETIC ANOMALY DETECTION (MAD) OPERATIONS.		5. TYPE OF REPORT & PERIOD COVERED 9 Final report, on an NRL problem.
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) 10 M. Nisenoff, J.J. Kennedy and A.W. Webb		8. CONTRACT OR GRANT NUMBER(s) N00014-75-WR0104 N00014-75-WR0105 61153N
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Research Laboratory Washington, D.C. 20375		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NRL Problem P05-04 RR011-11-01 R9103
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Arlington, Virginia 22217		11 12. REPORT DATE June 1977
		13. NUMBER OF PAGES 42
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) 12 42p.		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited. 16 RRφ1111

14 NRL-MR-3534 17 RRφ11101

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Magnetic Anomaly Detection (MAD)
Superconductivity
Superconductive device
Superconducting solenoid
Magnetic calibration source

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A superconducting solenoid was designed, built and tested to demonstrate the feasibility of using this type of device as a calibration source for Magnetic Anomaly Detection operations. A description of the dewar system required for such a solenoid is also described. The magnetic moment of the solenoid which was 48 cm in outer diameter, was ~~1.5×10^5~~ 150,000 ampere-turns-meter squared.

251 950 LB

CONTENTS

I. INTRODUCTION 1

II. DESIGN 1

III. ACQUISITION OF MAGNETIC CALIBRATION SOURCE 8

IV. TEST AND EVALUATION OF MAGNETIC CALIBRATION SOURCE 9

V. SUMMARY AND RECOMMENDATIONS 23

ACKNOWLEDGEMENTS 25

REFERENCES 25

APPENDIX A-1 26

APPENDIX A-2 28

APPENDIX A-3 30

APPENDIX A-4 33

SUPERCONDUCTING MAGNETIC CALIBRATION SOURCE FOR MAGNETIC ANOMALY DETECTION (MAD) OPERATIONS

I. INTRODUCTION

Objective

In order to demonstrate the feasibility of new concepts in Magnetic Anomaly Detection (MAD), there is a need for a calibrated target to simulate a typical MAD target. In this report, we describe the design, construction and performance of a superconducting solenoid which can be used for this purpose.

Superconducting technology is nearly ideal for this application. The magnetic moment of a superconducting solenoid can be calculated with good precision and the axis of the moment is known to lie along the geometric axis of the solenoid. By varying the current through the solenoid, the magnitude of the magnetic moment can be varied from (near) zero to some maximum value. The superconducting solenoid and the dewar system, which is necessary to provide the required cryogenic environment for the solenoid, can be small enough so that the calibration source can be made relatively mobile.

II. DESIGN

1. Preliminary Considerations

The magnetic dipole moment of a uniformly wound solenoid of length 2b, inner radius a_1, outer radius a_2, is given by the following relation (see Appendix A-3),

$$\mathcal{M} = \frac{N \Lambda I}{10} \quad \frac{a_1^2}{3} \quad \frac{a_2^3 - a_1^3}{a_2 - a_1}$$

Note: Manuscript submitted May 25, 1977.

where $\mathcal{M}$ = magnetic dipole moment

N = total number of turns in solenoid

I = current in windings

a_1 = inner radius

a_2 = outer radius

Λ = winding filling factor

$$= \frac{\text{cross-sectional area of windings that carry current}}{\text{total cross-sectional area of solenoid winding}}$$

In a real situation, the size of the solenoid cannot be made arbitrarily large as the solenoid would become too unwieldly and, in addition, the construction of a cryogenic container would be very difficult and rather expensive. Therefore, in practice, the diameter of a superconducting solenoid to be used as a magnetic dipole source should be made as large as possible consistent with size restrictions imposed by price and by the intended application.

During preliminary design considerations performed at NRL, the design goal for the magnetic moment of the solenoid was set at 3×10^5 ampere-turns-meter^2. To minimize cost during this initial feasibility exercise, it was further decided that commercially available niobium-titanium wire would be used. Although other types of superconducting wire, such as niobium-tin (Nb_3Sn) and vanadium-gallium (V_3Ga), have higher reported values for critical current density than does NbTi, these materials are just coming out of the laboratory and thus are much more expensive than NbTi wire. In addition these materials are much less ductile and greater care would have to be taken in fabricating the coil.

Initial designs were based on the use of 25 mil (0.65 mm.) NbTi wire containing 132 filaments and having a copper to superconductor ratio of 2.6:1. According to the literature of several wire manufacturers, the critical current vs. magnetic field characteristics of this type of wire is as follows:

Magnetic Field	Critical Current
80 KG	70 Amperes
60	110
40	165

This wire weighs approximately 1.7 pounds per 1000 feet and the calculations were based on a wire length $\mathcal{L}$ of 46,000 feet (1.40×10^4 km.) which weighs about 80 pounds.

In Fig. 1 the results of these calculations are summarized. The magnetic moment is shown as a function of the length to diameter ratio (β) of the solenoid for four different dewar sizes. In Fig. 2 the values of $\mathcal{M}$ for a value of β=0.5, corresponding to the maxima in the curves in Fig. 1, are plotted as a function of the square of the diameter of the dewar.

These results were obtained using the rated value of the critical current for the calculated magnetic field at the windings. However, in practice as a safety factor, magnets are usually operated at about 2/3 of their rated current. Using this factor, a solenoid with a magnetic moment of 3×10^5 ampere-turns-meter^2 would require a 30 inch (76.2 cm) inner diameter dewar. It was felt that a 20 (50.8 cm) nch ID dewar was probably as large as one would want to handle and thus the maximum value of moment for this system was set at about 1.5×10^5 ampere-turns-meter^2.

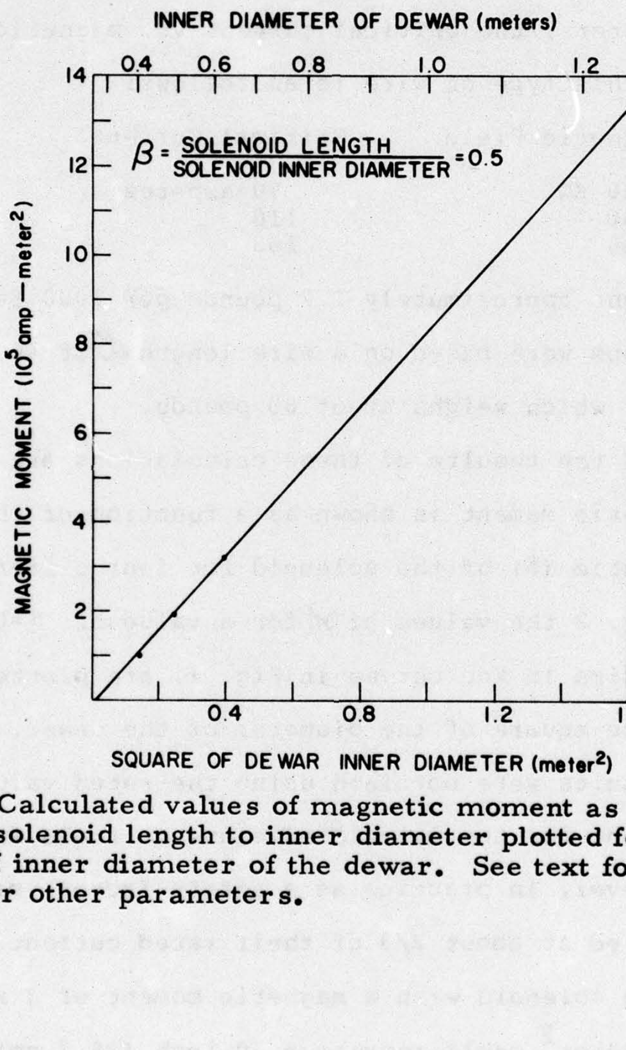

Fig. 1 - Calculated values of magnetic moment as function of ratio of solenoid length to inner diameter plotted for several values of inner diameter of the dewar. See text for assumed values for other parameters.

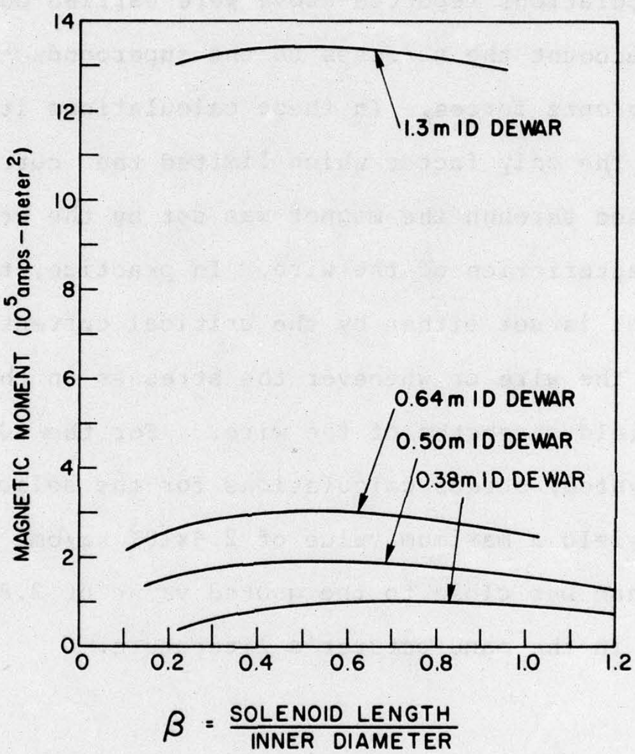

Fig. 2 - Calculated values of magnetic moment as function of dimensions of dewar for a fixed value of the ratio of solenoid length to inner diameter. See text for values assumed for other parameters.

The parameters of the solenoid and dewar according to these calculations are summarized in Table I.

The calculations reported above were carried out without taking into account the stresses on the superconducting wire due to the Lorentz forces. In these calculations it was assumed that the only factor which limited the current that could be passed through the magnet was set by the critical current characteristics of the wire. In practice, the limit to the current is set either by the critical current characteristics of the wire or whenever the stresses on the wire exceed the yield strengths of the wire. For the 20 inch system (50.8 cm.) system, stress calculations for the selected operating current yield a maximum value of 2.4×10^3 kg/cm^2 which is smaller than but close to the quoted value of 2.8×10^3 kg/cm^2 given in the manufacturer's literature.

TABLE I

	NRL Design Considerations	Vendor Design Parameters
Length (2b)	20 cm	17.2 cm
Outer diameter ($2a_2$)	46.7 cm	47.8 cm
Inner diameter ($2a_1$)	42.2 cm	37.6 cm
$\alpha = 2a_2/2a_1$	1.10	1.27
$\beta = 2b/2a_1$	0.47	0.46
Length of wire	46,000 ft.	48,400 ft.
Weight of Wire	80 lbs.	82 lbs.
H_{Center}	33,200 gauss	28,600 gauss
$H_{winding}$	54,300 gauss	47,800 gauss
$I_{critical}$	121.5 amps	142 amps
$I_{operate}$ (= 2/3 $I_{critical}$)	81 amps	96 amps
$\mathcal{M}$	$1.5\text{x}10^5$ amp-turns-meter^2	
Weight of Solenoid Including Thimble	150 pounds (est)	147 pounds
Weight of Support Structure	100 pounds (est)	152 pounds
Weight of Dewar	250 pounds (est)	450 pounds
Total Weight of System	500 pounds (est)	749 pounds
Overall Dewar Height	-	185 cm.
Outer Diameter of Dewar	-	80 cm.

III. ACQUISITION OF MAGNETIC CALIBRATION SOURCE

Request for Proposal (RFP) was issued for the construction and delivery to NRL of a system consisting of:

(1) a superconducting solenoid, approximately 20 inches (50.8 cm) in outer diameter, which had a magnetic dipole moment of 1.5×10^5 ampere-turns-meter squared,

(2) a persistent switch for the solenoid,

(3) an electronic power supply to energize the solenoid,

(4) a superinsulated dewar to provide the required cryogenic environment for the solenoid, which would have a helium boil off rate of less than 3 liter/hr when the solenoid was positioned in the dewar,

(5) a liquid helium level gauge indicator and control unit, and

(6) a header and support assembly to support the solenolid in the dewar with provisions to orient the solenoid either vertically or horizontally.

The RFP went out for bids and a contract was awarded to the Magnetic Corporation of America, Waltham, Mass. 02154. The specifications of the system as finally designed by MCA are included in Table I for comparison with the results of the preliminary calculations and design study performed at NRL. In their design calculations MCA concluded that there were serious stress problems in the magnet and therefore could not use a simple solenoid design but had to reinforce the solenoid by the use of strength members wound alternatively with the current carrying superconducting wire.

IV TEST AND EVALUATION OF MAGNETIC CALIBRATION SOURCE

1. Introduction

The Magnetic Calibration Source built by The Magnetic Corporation of America is shown assembled in Fig. 3 and the superconducting solenoid and solenoid support structure are shown in Fig. 4.

In evaluating the performance of this system, the following characteristics were examined:

(1) the procedure for cooling the solenoid and the helium boil-off rate of the system with the solenoid energized,

(2) procedures for charging and discharging the solenoid,

(3) the magnetic moment of the solenoid as determined from the magnetic field vs distance characteristics,

(4) the possible degradation of the solenoid performance after quenching, and

(5) the possibility of energizing the solenoid using standard automobile batteries.

2. Dewar Performance

The dewar and solenoid were precooled by filling the dewar with liquid nitrogen until the coil was completely immersed. This required about 320 liters of liquid nitrogen. The dewar system probably reached thermal equilibrium after 24 hours, but usually, two or three days were allowed for the precooling procedure.

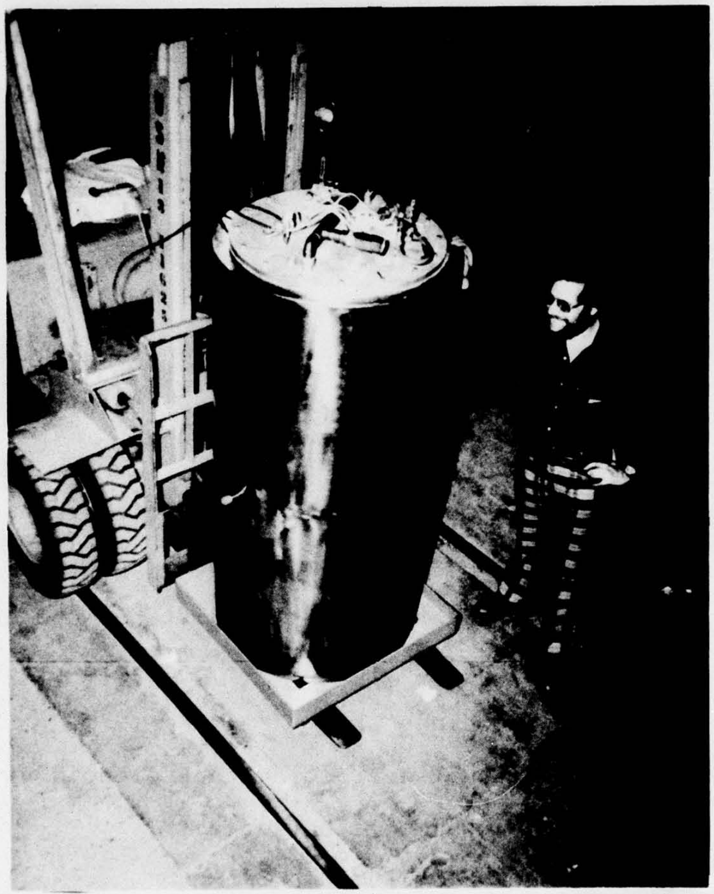

Fig. 3 - Photograph of Magnetic Calibration Source.

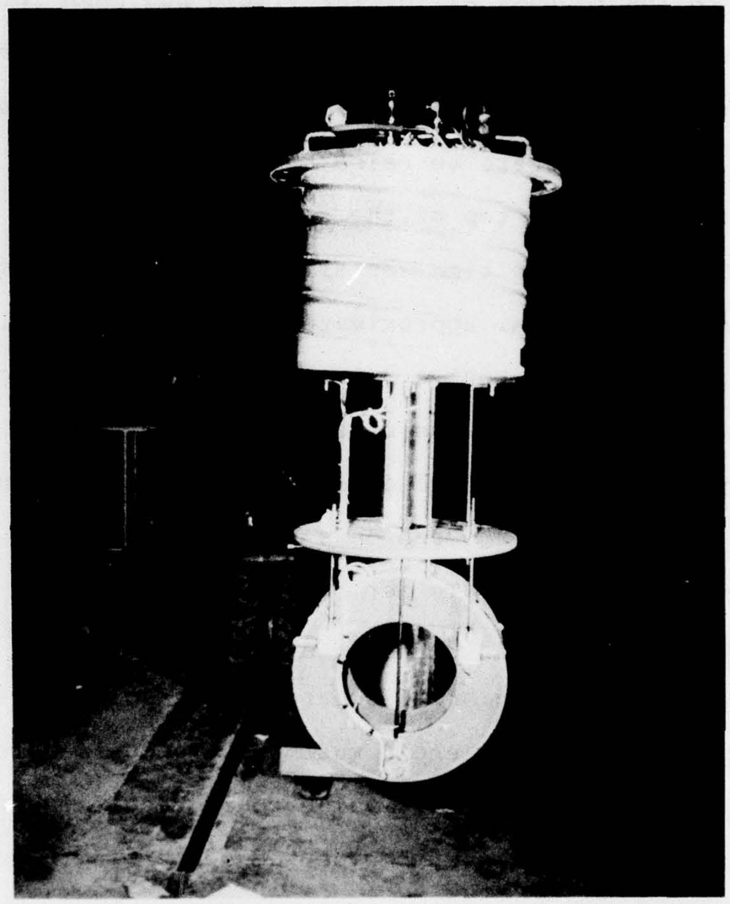

Fig. 4 - Photograph of Magnetic Calibration Source with dewar removed. The superconducting solenoid, current leads, support member, neck plug (to reduce convection and radiation losses through throat of dewar), and header plate are seen.

After the precooling was completed, the remaining liquid nitrogen was transferred out of the dewar with great care taken to insure that all of the liquid had been removed. Liquid helium was then transferred. About 70 liters of helium was required to cool the system from 77 K to 4 K and an additional 150 liters to raise the helium level to about 6 inches above the top of the solenoid.

After adequate time was allowed for the system to reach thermal equilibrium, approximately 6 hours, the helium boil off rate was about 1.3 liters of liquid per hour. The boil-off rate was essentially the same whether the solenoid was de-energized or driven at rated current. (This boil-off rate corresponds to a gas flow rate of 930 liters of gas at NTP per hour.)

3. Procedures for Charging the Solenoid

During charging and discharging the solenoid, the voltage across the solenoid must not exceed 14 volts. Voltages in excess of this value could result in either electric field breakdown between adjacent windings of the solenoid, or during discharge, excessive voltages could damage the power supply.

The persistent switch across the solenoid is opened, that is, driven into the non-superconducting state, by passing a current in excess of 100 milliamperes through the heater surrounding the switch.

The magnet can be charged by operating the power supply in the constant voltage mode, that is, by setting the voltage and allowing the current to build up at a rate

determined by the L/R time constant of the solenoid and leads. With the power supply set to 10 volts, the initial charging rate is 9.3 amps per minute and the rated current of 86.8 amps is reached in 9.4 minutes. (As the current approaches the rated value of 86.8 amps, care must be taken that the current does not exceed the rated value as damage to the solenoid might result.)

After the desired current has been achieved, the persistent switch across the solenoid can be closed. The voltage across the solenoid, and thus also across the persistent switch, is monitored until it drops below about 0.3 volts. The heater current is then reduced to zero and the switch will close in about 2 minutes. Once the switch has closed, the power supply can be turned off; the solenoid will be in the persistent mode and the magnetic moment of the solenoid will remain constant.

Under normal conditions, for safety sake, it is prudent to employ a less drastic charging procedure. The current is ramped with the supply set to 10 volts until the current reaches about 50 amps, the voltage is then reduced to 6 volts until the current reached 70 Amps and, finally reduced to about 4 volts until the rated current of 86.8 amperes is reached. This charging procedure minimizes the possibility of accidental quenching of the solenoid.

4. Determination of the Magnetic Moment of the Solenoid

The magnetic moment of the solenoid was determined by measuring the magnetic field as a function of distance along the geometric axis of the solenoid. A rotating coil magnetometer

was used to measure the magnetic field. A plot of magnetic field along the axis of the solenoid as a function of distance while the solenoid was operating at the rated current of 86.8 amperes is shown in Fig. 5. The solid line is the calculated variation of the field as a function of distance for an ideal, uniformly wound solenoid with the same dimensions - length, inner radius and outer radius - as the real solenoid. The only adjustable parameter, which could not be measured directly for this particular orientation of the solenoid in the dewar, was the magnetic field at the geometric center of the solenoid. The same data plotted logrithmically against distance is shown in Fig. 6. The solid line in Fig. 6 which has a slope of minus 3 is a fairly good fit to the data. The slight deviations from the expected dependence at large distances is probably due to stray fields from magnetized iron reinforcement rods in the concrete floor below the experiment, or to uncertainties in field measurements as the measured fields were comparable to instrument noise.

The magnetic moment of the solenoid at rated current deduced from the data plotted in Figs 5 and 6 had a value of 1.44×10^5 ampere-turns-meter^2. This value is about 4 percent low compared to the design value. However, since our magnetometer had not been calibrated prior to these measurements, this discrepancy will be assumed to be within experimental uncertainties until more precise measurements can be performed.

In order to investigate the magnitude of any hysteresis

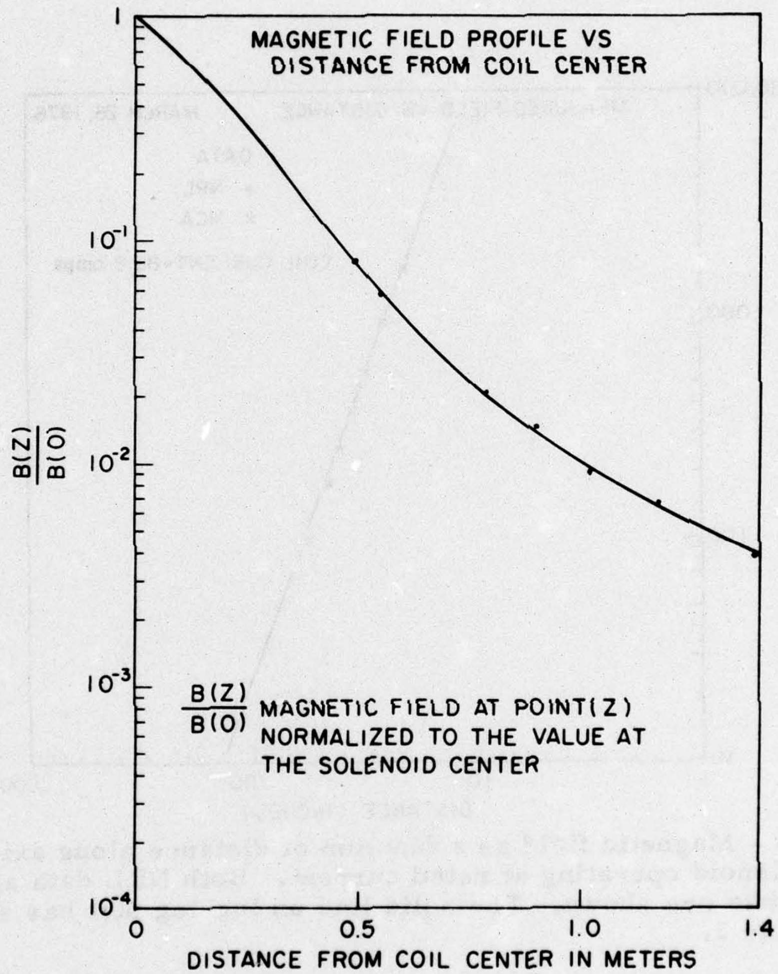

Fig. 5 - Magnetic field at a point along the axis normalized to the magnetic field at the center of the solenoid as a function of distance along axis. The solid line is the calculated variation for an ideal uniformly wound solenoid with the same parameters (dimensions, number of turns, etc.) as the actual solenoid.

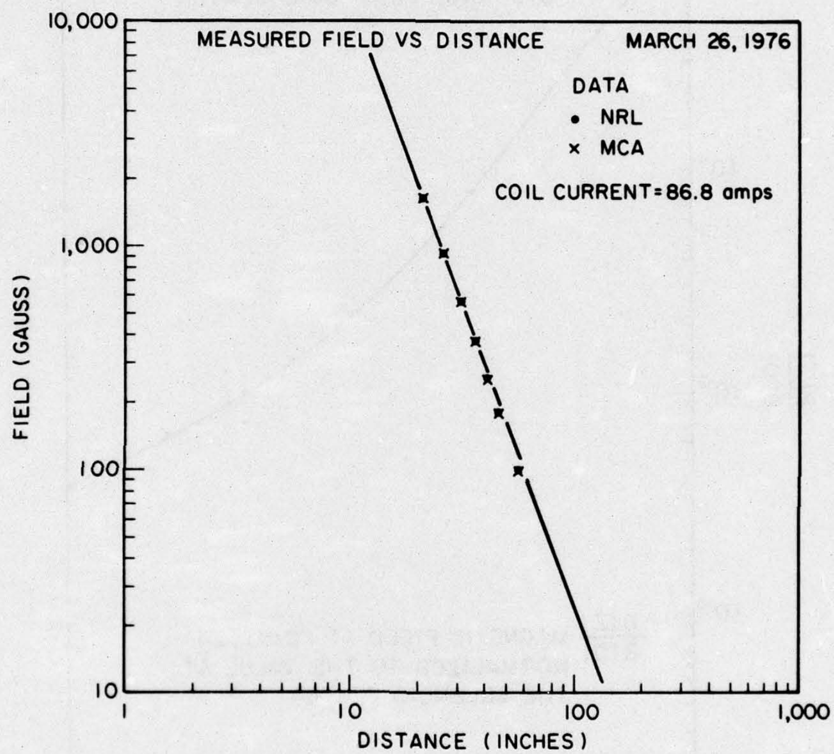

Fig. 6 - Magnetic field as a function of distance along axis for solenoid operating at rated current. Both NRL data and MCA data are shown. The solid line on log-log plot has slope of minus 3.

exhibited by the solenoid, the magnetic field at a fixed point was measured as the solenoid current was first increased from zero to 60 amperes and then reduced to zero. The data plotted in Fig. 7 clearly shows that the field at a given value of current was detectably lower for <u>in</u>creasing current than for <u>de</u>creasing current. The intercepts on the field axis for zero current is about plus or minus 10 gauss which implies that the solenoid can have a <u>current independent</u> moment of the order of plus/minus 7.5×10^2 ampere-turns-meter^3. This current independent magnetic moment undoubtedly arises from magnetic flux trapped in the windings of the solenoid. As is true in all cases of hysteresis, the exact magnitude and sign of the "remnant" moment depends very critically on the magnetic history of the solenoid after it had been cooled into the superconducting state. (The magnetic history of the solenoid prior to the taking of the data was not documented.) This remnant magnetic moment means that the error in knowing the magnetic moment using solenoid current alone is probably of the order of one percent at rated current, with the uncertainty becoming progressively larger as the solenoid current decreases. If the magnetic moment of the solenoid needs to be known to a precision of better than one percent for current less than about one-half rated current, actually field measurements close to the dewar would have to be made after the solenolid current has been established.

5. <u>Discharging the Solenoid</u>

Before the solenoid can be discharged, the persistent switch must be opened. This is accomplished by connecting the electronic power supply to the terminals of the solenoid,

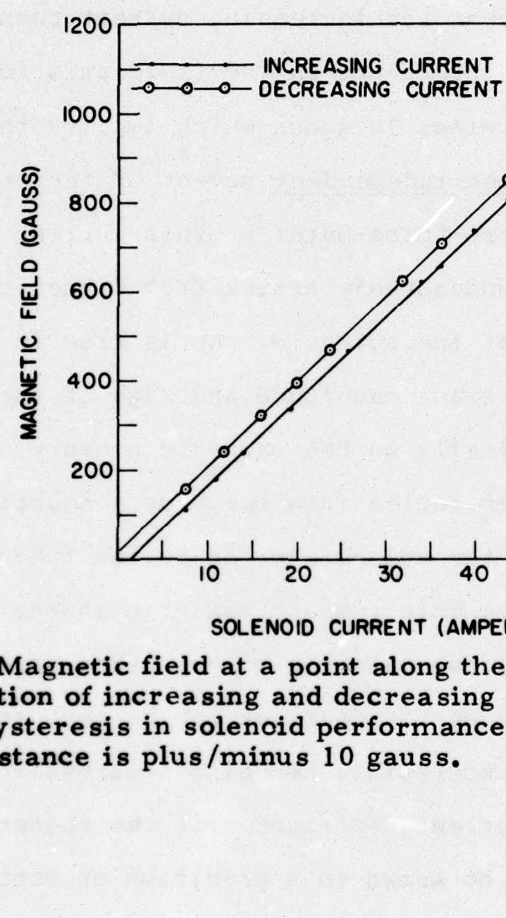

Fig. 7 - Magnetic field at a point along the axis of the solenoid as a function of increasing and decreasing solenoid current illustrating hysteresis in solenoid performance. The "remnant field" at this distance is plus/minus 10 gauss.

turning the supply on and raising the output current up to the value equal to the current flowing in the solenoid. The compliance voltage is set at about 2 volts. A current of greater than 100 milliamps is applied to the heater in the persistent switch. About 1 minute is required for the switch to open.

Once the persistent switch is opened, the current limit of the power supply is reduced until the back voltage across the solenoid is of the order of 10 volts. This mode of de-energizing the solenoid requires about 44 minutes. Since the inductance of the solenoid is 54.7 henries, this decay time implies that the sum of the lead resistance and the internal resistance of the power supply is about 0.066 ohms. The resistance of the current leads and the internal resistance of the power supply can be calculated from the equation

$$R = \frac{L}{\tau} \ln \left(I_{final}/I_{initial}\right)$$

where R is the total resistance, L is the inductance of the solenoid, $I_{initial}$ and I_{final} are the initial and final values of the decaying current in amperes and τ is the time in seconds required for the current to decay from $I_{initial}$ to I_{final}.

These long discharge times are not very convenient. Inspection of the equation given above indicates that to decrease the decay time τ the resistance in the circuit should be made larger. Therefore, two one-ohm 1,000 watt rheostats in parallel were placed in series with the magnet. With the

rheostats set to zero resistance, the solenoid current was allowed to decay from maximum rated current down to about 60 amperes. At this point about 0.18 ohms was added to the circuit and the current decay rate was raised to about 9.6 amps per minute. When the current had decreased below 30 amps, additional resistance was added to the circuit increasing the decay rate to about 15 amps per minute. This procedure was repeated until the current had been reduced to zero, always maintaining the back voltage across the solenoid at values below about 10 volts. When this procedure was optimized, it was possible to discharge the solenoid in about 17 minutes, compared to the "natural" discharge time of 44 minutes.

6. Battery Power Supply

Since the Magnetic Calibration Source will be used at relatively remote sites where electrical power may not be available to operate the electronic power supply, tests were made to see if the solenoid could be energized using 6 volt automobile batteries. Two batteries in parallel were connected to the solenoid and the current in the solenoid initially began to increase at a rate of 6.3 amps/minute, consistent with the rate observed with the electronic power supply at the same voltage setting. After 850 seconds, the current through the magnet was 72 amps with a slightly decreased charging rate as the terminal voltage of the batteries had fallen to 4.9 volts. The solenoid was put into the persistent mode, the batteries disconnected and a 0.1 ohm dump resistor connected across the solenoid. The persistent switch was then opened and the solenoid current was allowed to decay. After 950 seconds, the current was below 5 amps.

Although the test cited above was terminated before the solenoid was fully charged, it would appear from the arguments given below, that an automobile battery ought to be adequate to fully energize the solenoid. The stored energy in the solenoid is given by

$$W = \frac{1}{2} L I^2$$

where L is the inductance of the solenoid in henries and I is the current in amperes. For the solenoid, the stored energy at rated current is

$$W = \frac{1}{2} (54.7) (86.8)^2 = 216 \text{ kilojoules.}$$

Fully charged 6 volt lead batteries are rated at 70 amp-hours which corresponds to a stored energy of

$$W = E I t = (6) (70) (3600) = 1{,}512 \text{ kilojoules}$$

Thus if the experiment cited above had been allowed to reach completion the stored energy in the batteries would have been adequate to energize the solenoid to its rated current of 86.8 amps. Since the charging rate in the experiment was falling as the charging proceeded, the time required for reaching a desired current can be decreased by placing additional batteries in parallel.

7. Quench Test

In one possible mode of operation, the fully energized solenoid may be mounted on a mobile platform. Mechanical motion tends to encourage the probability of magnet quenching. Therefore it was written into the contract that, if the sole-

noid when fully energized should accidentally quench it must be capable of achieving rated current on the first energizing subsequent to the quench; that is, the solenoid should be capable of withstanding a quench without requiring any "training" to achieve rated current after the quench.

The usual procedure to quench a superconducting magnet is to exceed its rated current. The manufacturer strongly recommended against this practice as damage might be done to the solenoid. Therefore, a different procedure had to be devised.

The solenoid was energized to rated current of 86.8 amps and placed in the persistent mode. A tube was then inserted through one of the openings in the top plate of the dewar and penetrated until it touched the bottom of the dewar. Helium gas at room temperature was passed through the tube resulting in the evaporation of liquid helium. This procedure was followed for a period of time until the temperature of the solenoid began to rise causing the solenoid to quench. As soon as the "blow-off" stopped, about 30 liters of additional liquid helium was transferred into the dewar. The solenoid current was raised in a cautious manner to 86.8 amps. The magnetic field at a distance was the same as before the quench. The test was repeated successfully a number of times during several different coolings of the system.

V. SUMMARY AND RECOMMENDATIONS

A superconducting solenoid system has been built which can be employed as a Magnetic Calibration Source for evaluating the performance of various types of Magnetic Anomaly Detection (MAD) systems. Tests have shown that:

(1) a Magnetic Calibration Source with a maximum magnetic moment of 1.5×10^5 ampere-turns-meter^2 can be constructed with a reasonable system weight and size,

(2) the magnetic moment at rated current can be known to a precison of better than one percent,

(3) there is a remnant magnetic moment presumably due to trapped flux in the niobium-titanium wire used in winding the solenoid which can be of the order of 10^3 ampere-turns-meter^2.

(4) for operation in remote locations, the superconducting solenoid can be energized to rated current using two (or possibly three) standard six volt automobile lead storage batteries,

(5) the drawbacks of the Magnetic Calibration Source built during this program are the overall system weight (about 700 pounds), the quantity of liquid nitrogen needed to precool the system (300 liters,

approximately one half of which can be recovered) and the quanity of liquid helium required to fill the dewar (approximately 200 liters).

In designing and building subsequent Magnetic Calibration Sources, the design should be optimized to reduce the size and weight of the system and the quantities of cryogens required. These goals might be implemented by:

(1) the use of either Nb_3Sn or V_3Ga multifilamentary wire, which have reported values for overall wire current densities about an order of magnitude greater than niobium-titanium wire would reduce the amount of wire required,

and

(2) the use of a pancake shaped or toroidal-shaped dewar which closely follows the contours of the solenoid would reduce the weight of the dewar and the amounts of cryogens required.

With these improvements, the total system weight could probably be reduced to about 300 pounds, the amount of cryogens required halved, and thus a system more suitable for mobile applications than the present system could be obtained.

ACKNOWLEDGEMENTS

We would like to thank L. S. Weinman and V. O. Johnson for their assistance in carrying out some of the preliminary calculations. We would also like to acknoweldge D. U. Guser and R. A. Hein for several helpful discussions on the design of large diameter pancake solenoids.

REFERENCES

"Solenoid Magnet Design", D. B. Montgomery (Wiley-Interscience, New York, 1969).

"Table of Ratio D_0/B_{max} for Magnets with Rectangular Cross-Section and Uniform Current Density", M. V. Garrett (Oak Ridge National Laboratory Report ORNL 3972, August 1966; available also from National Technical Information Service, Springfield, VA 22161).

"Some Practical Consequences of Platform Instability for a Superconducting Gradiometer MAD System". J. R. Clement, E. M. Compy and M. Nisenoff; NRL Memorandum Report 2648 (October 1973).

APPENDIX A-1

The problem of units is one of the most disturbing and infuriatnig parts of carrying out calculations in electricity and magnetism. Not only are the sets of units based on MKS units as well as on C.G.S. units but there are additional variations where the quantity 4π either appears or does not appear in "familiar" equations.

In the report, the CGS gaussian equations and units are employed as most of the references and books in the physics literature are written in these units.

<u>MAGNETIC INDUCTION B</u> - In the MKS system, B is expressed in tesla while in the gaussian system B is expressed in gauss. The conversion factor is

$$1 \text{ Tesla} = 10^4 \text{ gauss}$$

<u>MAGNETIC MOMENT</u> - The units of $\mathcal{M}$ are current.turns. area and thus in MKS $\mathcal{M}$ is expressed in ampere.turns.meter^2 while in gaussian units $\mathcal{M}$ is expressed in abamp.turns.cm^2. The conversion factor is

$$1 \text{ Amp m}^2 = 10^3 \text{ Abampere.cm}^2$$

In gaussian units abamp/cm is equivalent to a gauss so we can also express in units of gauss cm^3

$$\begin{aligned} 1 \text{ Ampere meter}^2 &= 10^3 \text{ Abampere cm}^2 \\ &= 10^3 \text{ gauss cm}^3 \\ &= 3.53 \times 10^{-2} \text{ gauss ft}^3 \\ &= 3.53 \times 10^3 \text{ gamma ft.}^3 \end{aligned}$$

The latter two equivalences on the right side are badly "mixed" units which are not acceptable to "purists" but are included as the last one, gamma ft ^{3}, is commonly used by the MAD community.

APPENDIX A-2

The magnetic moment of a current loop of N identical turns of wire is, by definition

$$\mathcal{M} = N.I.\ \text{Area}. \qquad \text{A-1}$$

If the loop is circular in cross-section, then Eq. A-1 can be written as

$$\mathcal{M} = NI\pi r^2 \qquad \text{A-2}$$

where r is the radius of the loop. If the windings have finite dimension, then Eq. A-2 can be written as

$$\mathcal{M} = I\,\pi\,\overline{N(r)r^2}$$

According to the Mean Value Theorem, the quantity $\overline{N(r)r^2}$ can be calculated as follows,

$$\overline{N(r)r^2} = \frac{1}{a_2 - a_1}\int_{a_1}^{a_2} N(r)r^2\,dr \qquad \text{A-3}$$

where a_2 is the outer radius of the solenoid and a_1 is the inner radius. If the winding density N(r) is uniform, that is, independent of radius r, than the above equation becomes

$$\overline{N(r)r^2} = \frac{N}{a_2 - a_1}\int_{a_1}^{a_2} r^2\,dr$$

$$= \frac{N}{3}\left(\frac{a_2^3 - a_1^3}{a_2 - a_1}\right) \qquad \text{A-4}$$

Using the definition $\alpha = a_2 / a_1$ Equation (A-4) becomes

$$\overline{N(r)r^2} = \frac{N}{3} a_1^2 \frac{\alpha^3 - 1}{\alpha - 1} \tag{A-5}$$

and the equation for the magnetic moment $\mathcal{M}$ of a thick loop with uniform winding density is given as

$$\mathcal{M} = \frac{N I \pi}{10} \frac{a_1^2}{3} \frac{\alpha^3 - 1}{\alpha - 1} \tag{A-6}$$

APPENDIX A-3

In this appendix the expression for the magnetic moment of a circular current loop of finite thickness will be derived from another viewpoint in order to see in what regime a solenoid of finite dimensions resembles an "ideal" magnetic dipole loop.

The axial component of the magnetic field produced by a solenoid at its geometric center point is given by (see Montgomery)

$$H(0,0) = j\Lambda \frac{4\pi\beta}{a_1 \; 10} \ln \frac{\alpha + (\alpha^2 + \beta^2)^{1/2}}{1 + (1+\beta^2)^{1/2}} \qquad \text{A-7}$$

where $H(0,0)$ = magnetic field at center of solenoid in gauss

j = current density in windings in amperes/cm^2

$= \dfrac{N\ I}{\Lambda 2b\ (a_2 - a_1)}$

a_1 = inner radius of solenoid (in cms)

a_2 = outer adius of solenoid (in cms)

$2b$ = length of solenoid (in cms)

α = a_2/a_1

β = $2b/2a_1$

$\Lambda = \dfrac{\text{cross sectional area of windings that actually carry current}}{\text{total cross sectional area of windings}}$

An alternate version of Equation (A-7) can be written as

$$H(0,0 = j\Lambda\, a_1 \left(\frac{4\pi\beta}{10}\right)\left[\sinh^{-1}\left(\frac{\alpha}{\beta}\right) - \sinh^{-1}\left(\frac{1}{\beta}\right)\right] \qquad \text{A-8}$$

The axial magnetic field at some point P at a distance Z away from the center of the solenoid but inside the solenoid (0   Z   b) can be written as

$$H(z,0) = j\Lambda a_1 \frac{2\pi}{10} \left\{ (\beta + z/a_1) \left(\sinh^{-1} \frac{\alpha}{\beta + z/a_1} - \sinh^{-1} \frac{1}{\beta + z/a_1} \right) + (\beta - z/a_1) \left(\sinh^{-1} \frac{\alpha}{\beta - z/a_1} - \sinh^{-1} \frac{1}{\beta - z/a_1} \right) \right\} \quad \text{A-9}$$

When the field point P is outside the solenoid (z>b) the expression for the axial component H(z,0) is given by

$$H(z,0) = j\, a \Lambda \frac{2\pi}{10} \left\{ (\beta + z/a_1) \left(\sinh^{-1} \frac{\alpha}{\beta + z/a_1} - \sinh^{-1} \frac{1}{\beta + z/a_1} \right) - (\beta - z/a_1) \left(\sinh^{-1} \frac{\alpha}{\beta - z/a_1} - \sinh^{-1} \frac{1}{\beta - z/a_1} \right) \right\} \quad \text{A-10}$$

Note that Equations A-9 and A-10 are nearly identical except for the sign of the second group of terms in the brackets on the right hand side of these equations.

To simplify this expression to see what the axial field is at a distance z (z>b) along the axis, the series expansion for $\sinh^{-1}$ can be used

$$\sinh^{-1} x = x - \frac{1}{2}\frac{x^3}{3} + \frac{1}{2}\frac{3}{4}\frac{x^5}{5} - \frac{1\ 3\ 5}{2\ 4\ 6}\frac{x^7}{7} +$$

With this expansion Equation A-10 reduces to

$$H(z,0) = 2 \frac{N\ I}{10} \pi a_1^2 \frac{1}{3} \frac{\alpha^3 - 1}{\alpha - 1} \frac{1}{z^3} \quad \text{A-11}$$

This expression is good to better than 1% when z > 10b. Since the axial component of the magnetic field of a magnetic dipole loop at a point along the axis is given by

$$H(z,0) = 2 \frac{\mathcal{M}}{z^3}$$

the thick solenoid will resemble a ideal magnetic dipole loop to better than 1% when the field point is at a distance z such that z > 10b (where 2b is the length of the solenoid) and the solenoid will appear to have a magnetic dipole moment such that

$$\mathcal{M} = \frac{N\ I}{10}\ \pi a_1^2\ \frac{1}{3}\ \frac{\alpha^3 - 1}{\alpha - 1}$$

consistent with the result obtained previously (see Equation A-6).

APPENDIX A-4

In this appendix, the procedure for calculating the magnetic moment of a solenoid that was employed for the preliminary design considerations performed at NRL will be outlined. These calculations were done assuming a fixed length of wire $\mathcal{L}$ of effective cross-sectional area $[\Phi]^2$ such that

$$[\Phi]^2 = \frac{(\text{radius of wire})^2}{\Lambda} \qquad \text{A-12}$$

where Λ is the "filling factor" of the coil windings.

Using the effective cross-section of the windings $[\Phi]^2$, the number of turns of wire in the windings can be determined as follows:

$$N = \frac{\text{cross-sectional area of winding}}{[\Phi]^2}$$

$$= \frac{2b(a_2 - a_1)}{[\Phi]^2} \qquad \text{A-13}$$

and the total length of the wire in the solenolid winding is given by

$$\mathcal{L} = N.(\text{average circumference of winding})$$

$$= N.2\pi\left(\frac{a_2 + a_1}{2}\right) = N\,(a_2 + a_1) \qquad \text{A-14}$$

combining Equations A-13 and A-14, one obtains,

$$\mathcal{L} = \frac{2\pi b}{[\Phi]^2}\,(a_2^{\,2} - a_1^{\,2}) \qquad \text{A-15}$$

$$= \frac{2\pi b\, a_1\, (\alpha^2 - 1)}{[\Phi]^2}$$

The expression for the axial component of the magnetic field at the geometric center of the solenoid H(0,0) given by Eq. A-8 can be rewritten using the definition for the effective current density in the

windings

$$j\Lambda = \frac{NI}{2b\,(a_2-a_1)} \qquad \text{A-16}$$

Using the relation between the total number of turns N in the windings and the length $\mathcal{L}$ of wire in the windings given by Eq. A-14, the effective current density, $j\Lambda$, can be written as

$$j\Lambda = \frac{I\,\mathcal{L}}{2\pi b\, a_1\, (\alpha^2-1)} \qquad \text{A-17}$$

and the expression for H(0,0) becomes

$$H(0,0) = \frac{I\,\mathcal{L}}{2\pi b\, a_1}\ \frac{1}{\alpha^2-1}\ \frac{4\pi\beta}{10}\left[\sinh^{-1}\left(\frac{\alpha}{\beta}\right) - \sinh^{-1}\left(\frac{1}{\beta}\right)\right] \qquad \text{A-18}$$

In a similar manner, the expression for the magnetic moment of the solenoid can be written in terms of the wire length using Equation A-14 yielding

$$\mathcal{M} = \frac{I\,\mathcal{L}\,a_1}{30}\ \frac{\alpha^3-1}{\alpha^2-1} \qquad \text{A-19}$$

Now that the expressions for H(0,0) and $\mathcal{M}$ have been expressed in terms of the wire length $\mathcal{L}$, the procedure used in the NRL design calculations can be outlined.

GIVEN: D = diameter of dewar

$\mathcal{L}$ = length of wire

$[\Phi]^2$ = effective cross sectional area of wire

ASSUME: $2a_2$ = outer diameter of solenoid ($2a_2 < D$).

CALCULATE: 2b = length of solenoid.

$$2b = \left\{ D^2 - (2a_2)^2 \right\}^{1/2}$$

CALCULATE: a_1 = inner radius of coil using Eq. A-15

$$a_1 = \left\{ a_2{}^2 - \frac{[\Phi]^2}{2\pi b} \right\}^{1/2}$$

CALCULATE: H(0,0)/I the center field of the solenoid per unit current through windings of solenoid using Eq. A-18.

$$\frac{H(0,0)}{I} = \frac{\mathcal{L}}{2\pi b\ a_1\ (\alpha^2 - 1)} \frac{4\pi\beta}{10} \left[\sinh^{-1} \frac{\alpha}{\beta} - \sinh^{-1} \frac{1}{\beta} \right]$$

CALCULATE: $\mathcal{M}$ the magnetic moment per unit current using Eq. A-19

$$\frac{\mathcal{M}}{I} = \frac{\mathcal{L}}{30} a_1 \frac{\alpha^3 - 1}{\alpha^2 - 1}$$

FROM TABLES: Use tables preapred by Garrett (plotted in Fig. A-1) to obtain values of G (α,β) which is ratio of magnetic field at the innermost windings of the solenoid to the center field of the solenoid H(0,0) for the given set of parameters $\alpha = a_2/a_1$ and $\beta = 2b/2a_1$

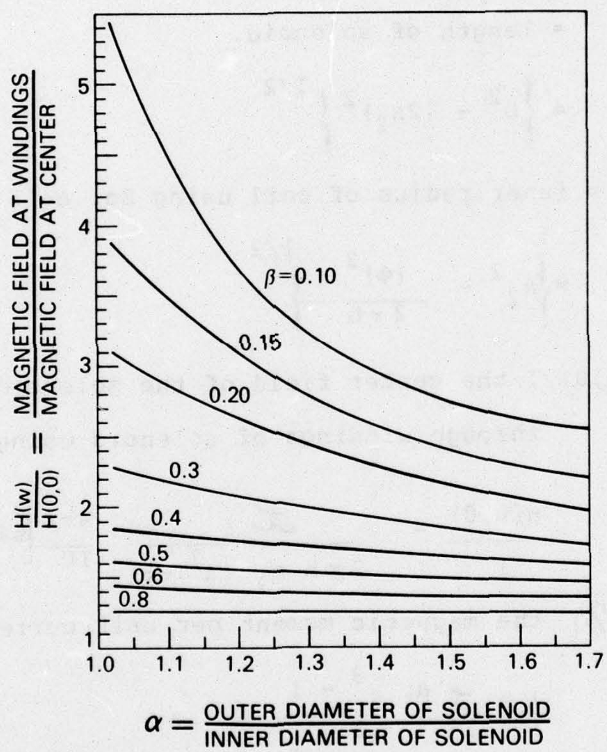

Fig. A-1 (a) Ratio of magnetic field at windings of a solenoid relative to magnetic field at center as function of ratio of outer to inner diameter of solenoid plotted for various values of the ratio of overall length of solenoid to inner diameter (β).

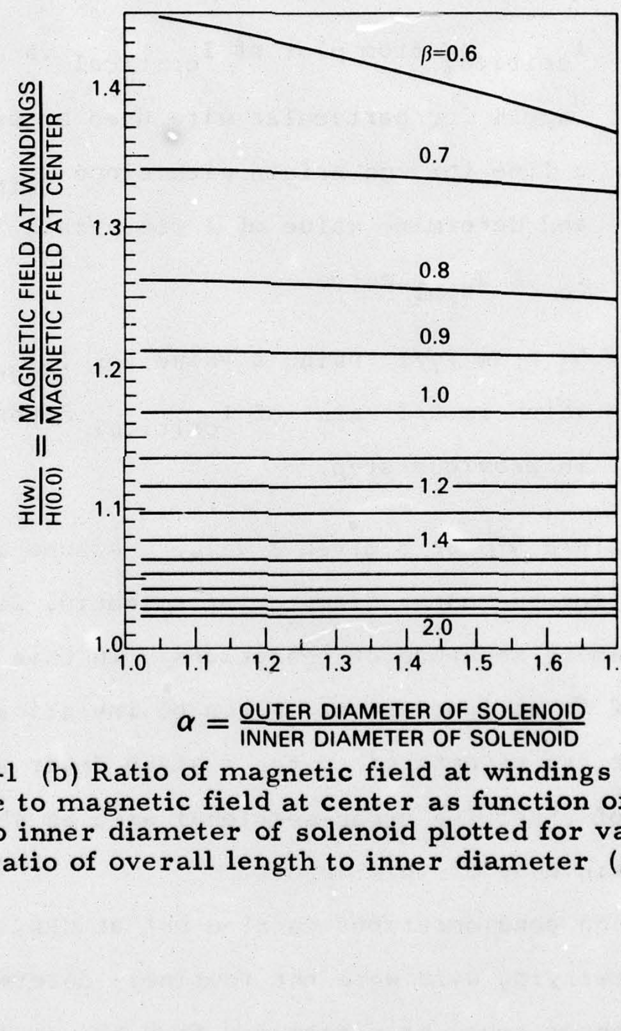

Fig. A-1 (b) Ratio of magnetic field at windings of solenoid relative to magnetic field at center as function of ratio of outer to inner diameter of solenoid plotted for various values of the ratio of overall length to inner diameter (β).

CALCULATE: The quantity

$$\frac{H_{winding}}{I} = \frac{H(0,0)}{I} \cdot \frac{H_{winding}}{H(0,0)}$$

DETERMINE: $I_{critical}$ from plot of $I_{critical}$ vs H_{applE} for particular wire used by drawing a line through origin with slope $H_{winding}/I$ and determine value of I where this line crosses I_c vs H_{appl} curve.

CALCULATE: $\mathcal{M}$ from $\mathcal{M}/I$ using a value for $I_{operate}$ which is 2/3 value of $I_{critical}$ determined in previous step.

This determines $\mathcal{M}$ for a given β value. Assume a different value for the outer diameter of solenoid, $2a_2$, and repeat the above sequence of operations. In this manner, the variation of $\mathcal{M}$ as function of β can be investigated and a maximum moment can be determined for a given dewar size, D, and $\mathcal{L}$ of wire of effective cross-sectional area as shown in Fig. 1 in the main body of this report.

In the design considerations carried out at NRL, stresses in the current carrying wire were not routinely determined. However, the stesses could be determined from the equation

$$\sigma = \frac{H\ I\ a_1}{A_c} \quad \frac{10^6}{9.8} \qquad \text{A-19}$$

where σ = stress in kilograms/cm^2

H = magnetic field acting on wire, in gauss

I = current in amperes

A_c = cross sectional area of wire in cm^2

This expression for the stress can be rewritten in terms of quantities determined in the calculation procedure as follows:

$$\sigma = \frac{H}{I} \quad \frac{(2\ I_{crit}/3)\ a_1}{A_c} \quad \frac{10^6}{9.8} \qquad \text{A-20}$$

If the calculated value of σ exceeds the critical stress of the wire, then, the operating current $I_{op} = 2\ I_{crit}/3$ must be reduced so σ is safely below the yield strength of the wire.